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"Small-Scale Dynamo Action in Primordial Halos"

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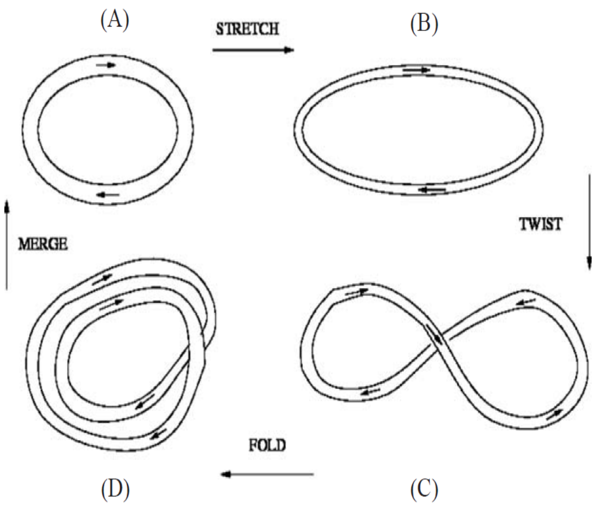
Content


“Small-Scale Dynamo & First Stars”


1. Introduction

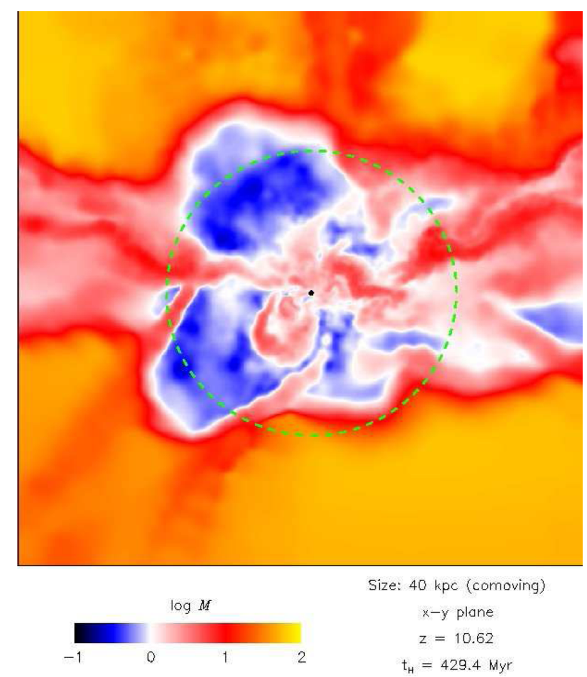
Small-Scale Dynamo

- **small-scale dynamo:** process which converts kinetic energy from turbulence into magnetic energy on short timescales



“Stretch-Twist-Fold Model”
[Brandenburg &Subramanian 2005]

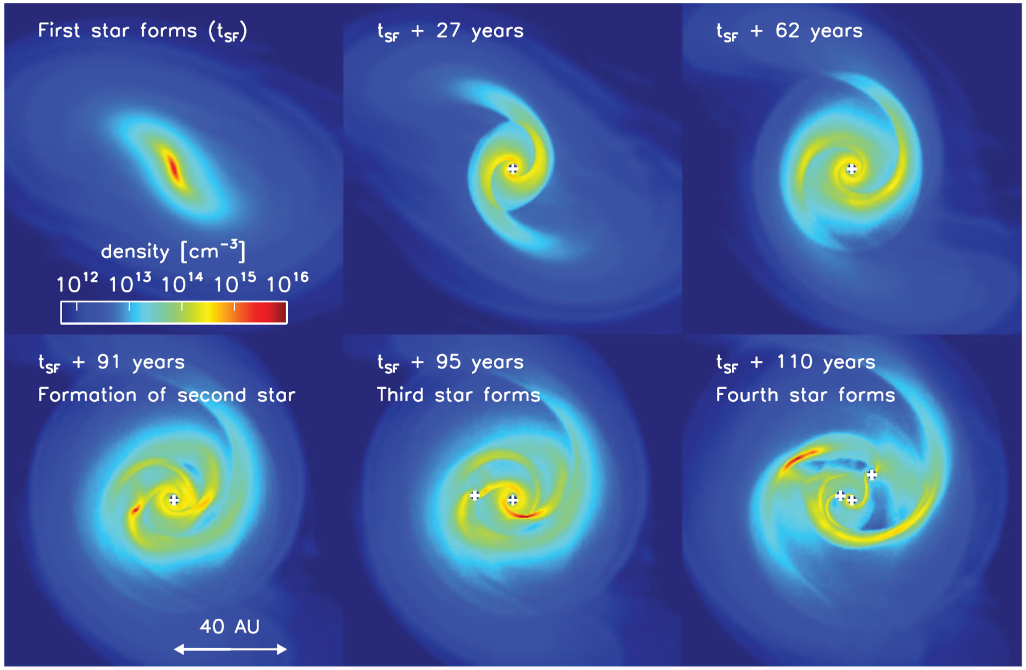
- first point in time, where dynamo can operate: **formation of the first stars** (first turbulent halos)



Mach number in Primordial Halo
[Greif et al. 2008]

Primordial Star Formation

- start at $z \approx 20$
- formation in dark-matter mini halos
- gas composition: H, He, Li, …
- temperature: $T \approx 1000$ K



[Clark et al., Science, 2011]

- further ingredients:
  - weak magnetic seed fields
  - turbulence: mixture of solenoidal and compressive modes

=> dynamo action in principle possible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2. Model for Primordial ISM

Magnetic Seed Fields

- **inflation:**

$$B \approx \left(10^{-34} - 10^{-9}\right)\ \mathrm{G}$$

[e.g. Turner & Widrow 1988]

- **phase transitions in the early Universe:**

$$B \approx 10^{-20}\ \mathrm{G}$$

[QCD phase transition, 10 Mpc comoving scale, e.g. Sigl et al. 1997]

- **battery processes:**

$$B \approx 10^{-20}\ \mathrm{G}$$

[Biermann battery, kpc scale, e.g. Xu et al. 2008]

=> extremely weak seed fields!

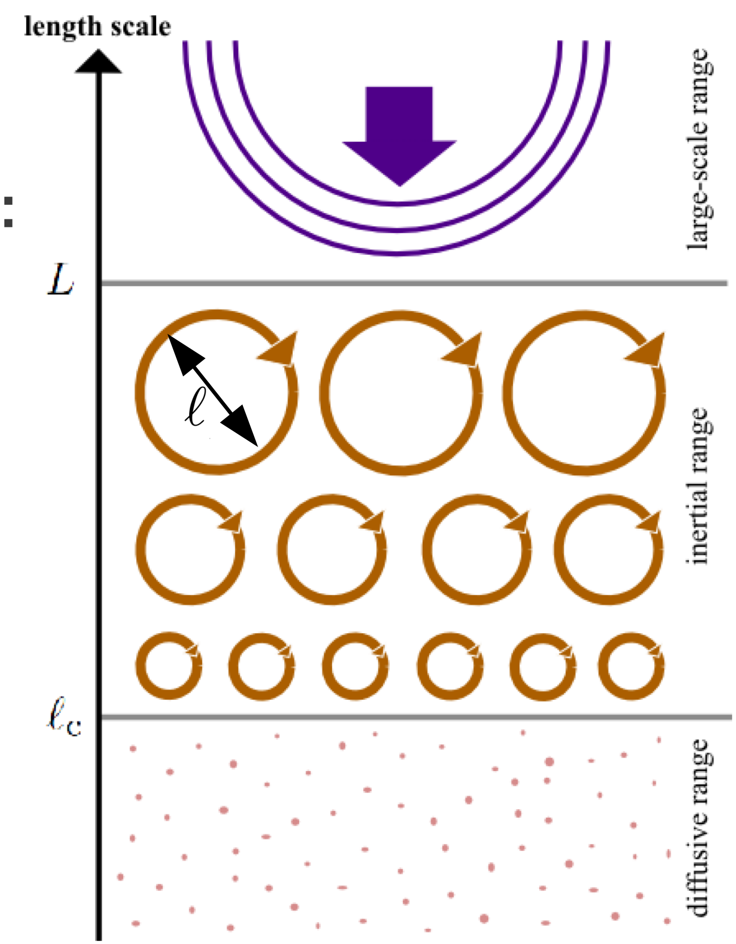
Turbulence

- turbulence in first star-forming halos is **driven by accretion**
- driving scale: Jeans scale
- **different types of turbulence:** relation in the inertial range:

$$\delta v(\ell) \propto \ell^{\vartheta}$$

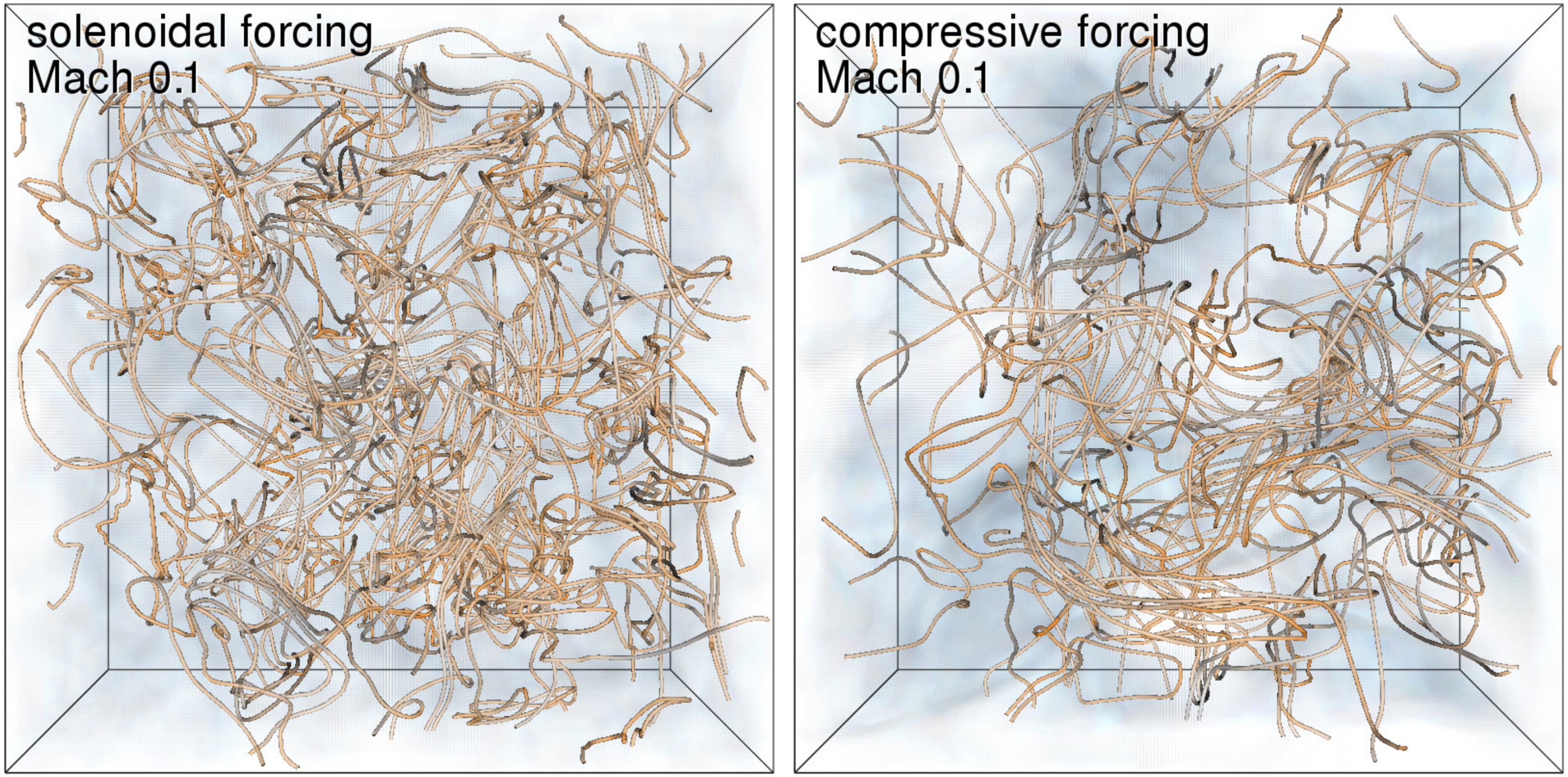
$$1/3 \leqslant \vartheta \leqslant 1/2$$

Kolmogorov (incompressible) — Burgers (highly compressible)



Different Forcing/Turbulence

effect of different types of forcing on the structure of magnetic field lines:



[Federrath et al., PRL, 2011]

=> solenoidal forcing twists field lines more effectively

The Velocity Field

- **separation of smooth and turbulent component:**

$$\mathbf{v} = \langle \mathbf{v} \rangle + \delta \mathbf{v}$$

- **properties of turbulent field $\delta \mathbf{v}$:**
  - isotropic and homogeneous
  - Gaussian random field with zero mean
  - delta-correlated in time

- **spatial two-point correlation function**
  = "measure for turbulent kinetic energy density":

$$\langle \delta v_i(\mathbf{r}_1, t) \delta v_j(\mathbf{r}_2, s) \rangle$$
$$= \left(\left(\delta_{ij} - \frac{r_i r_j}{r^2} \right) T_\mathrm{N}(r) + \frac{r_i r_j}{r^2} T_\mathrm{L}(r) \right) \delta(t - s)$$

transversal (T_N) longitudinal (T_L)

(note: We neglect helicity.)

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Model for T_{L} and T_{N}

- motivation: diffusion coefficient in inertial range
- **model for general turbulence:**

$$T_{\mathrm{L}}(r) = \begin{cases} \frac{VL}{3}\left(1 - Re^{(1-\vartheta)/(1+\vartheta)}\left(\frac{r}{L}\right)^2\right) & 0 < r < \ell_{\mathrm{c}} \\ \frac{VL}{3}\left(1 - \left(\frac{r}{L}\right)^{\vartheta+1}\right) & \ell_{\mathrm{c}} < r < L \\ 0 & L < r \end{cases}$$

[Vainsthein, Sov. Phys. JETP, 1982; Subramanian, ArXiv, 1997]

$$T_{\mathrm{N}}(r) = \begin{cases} \frac{VL}{3}\left(1 - t(\vartheta)Re^{(1-\vartheta)/(1+\vartheta)}\left(\frac{r}{L}\right)^2\right) & 0 < r < \ell_{\mathrm{c}} \\ \frac{VL}{3}\left(1 - t(\vartheta)\left(\frac{r}{L}\right)^{\vartheta+1}\right) & \ell_{\mathrm{c}} < r < L \\ 0 & L < r \end{cases}$$

[Schober et al., PRE, 2012]

(ℓ_{c} : viscous scale, L : scale of largest fluctuations,
$\mathrm{Re} = \frac{VL}{\nu}$: Reynolds number)

Chemistry & MHD Quantities

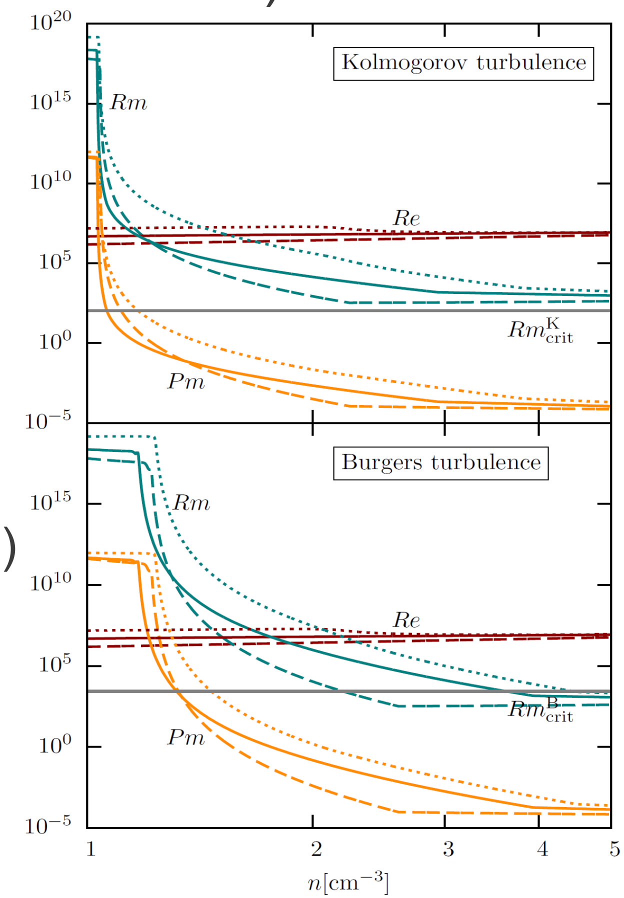
- **one-zone model of Glover & Savin (2009)**
  (with modification connecting collapse time and equation of state [Schleicher et al. 2009] and additional Li-chemistry [Bovino et al. 2011]):
  ~ 30 different species (atomic and molecular)
  ~ 400 different chemical reactions

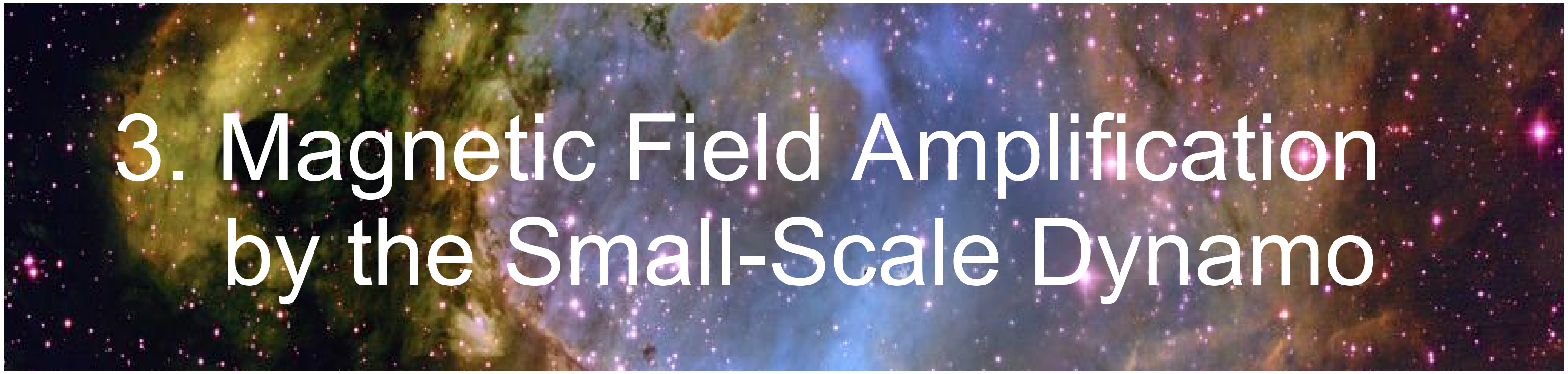
- **characteristic MHD quantities:**

$$\mathrm{Re} = \frac{VL}{\nu}$$ hydrodynamical Reynolds number

$$\mathrm{Rm} = \frac{VL}{\eta}$$ magnetic Reynolds number ($\eta = \eta_{\mathrm{Ohm}} + \eta_{\mathrm{AD}}(\mathbf{B})$)

$$\mathrm{Pm} = \frac{\mathrm{Rm}}{\mathrm{Re}} = \frac{\nu}{\eta}$$ magnetic Prandtl number




3. Magnetic Field Amplification
by the Small-Scale Dynamo

MHD-Dynamos – Mathematical Description

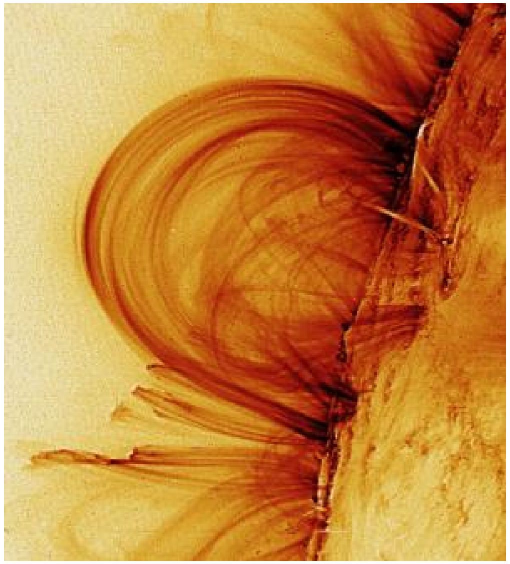
- idea: divide magnetic field into **mean and turbulent component**

$$\mathbf{B} = \langle \mathbf{B} \rangle + \delta \mathbf{B}$$

- put into **induction equation**:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{v} \times \mathbf{B} - \eta \nabla \times \nabla \times \mathbf{B}$$

=> evolution equations for mean and turbulent field (large-scale dynamo and small-scale dynamo)



Magnetic active region on the Sun
[credit: NASA, Trace Mission]

Kazantsev Theory: Model for Kinematic Growth

- put $\langle \delta B_i(\mathbf{r}_1, t) \delta B_j(\mathbf{r}_2, t) \rangle$(= "magnetic energy density") into induction equation

=> **Kazantsev equation** [Kazantsev, 1968]:

$$-\kappa_{\text{diff}}(r)\frac{\mathrm{d}^2\psi(r)}{\mathrm{d}^2 r} + U(r)\psi(r) = -\Gamma\psi(r)$$

$\psi(r)\mathrm{e}^{2\Gamma t}$ "magnetic energy density"

$\kappa_{\text{diff}}(r) = \kappa_{\text{diff}}(\langle \delta v_i(\mathbf{r}_1, t) \delta v_j(\mathbf{r}_2, s) \rangle, \eta)$ "mass"

$U(r) = U(\langle \delta v_i(\mathbf{r}_1, t) \delta v_j(\mathbf{r}_2, s) \rangle, \eta)$ "potential"

- can be solved with WKB approximation for large and small magnetic Prandtl numbers $\mathrm{Pm} = \mathrm{Rm}/\mathrm{Re}$

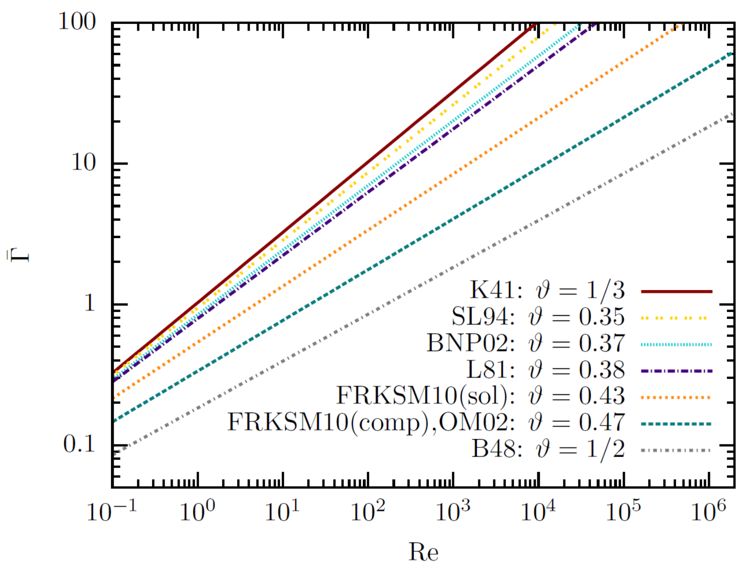
Critical Mag. Reynolds Number

- critical magnetic Reynolds number for small-scale dynamo action:
  set $\Gamma = 0$ in Kazantsev equation and solve for Rm
  $(\mathrm{Rm} = VL/\eta)$
- for Kolmogorov turbulence:
  $$\boxed{\mathrm{Rm} > 110}$$
- for Burgers turbulence:
  $$\boxed{\mathrm{Rm} > 2700}$$
- note: need high resolution in order to see dynamo in simulations

Growth Rate

- growth rate for large Pm:

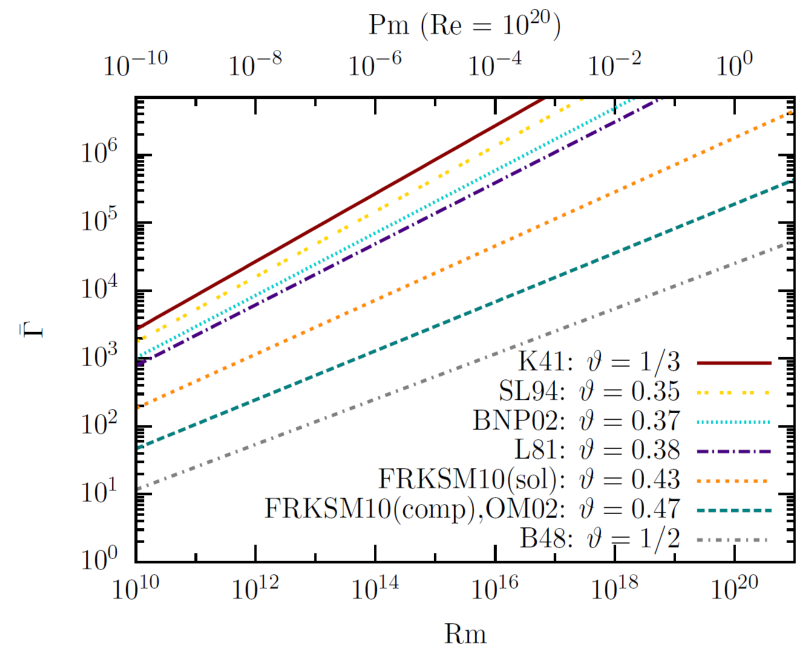
$$\Gamma \propto \mathrm{Re}^{(1-\vartheta)/(1+\vartheta)}$$



Schober et al., PRE, 2012

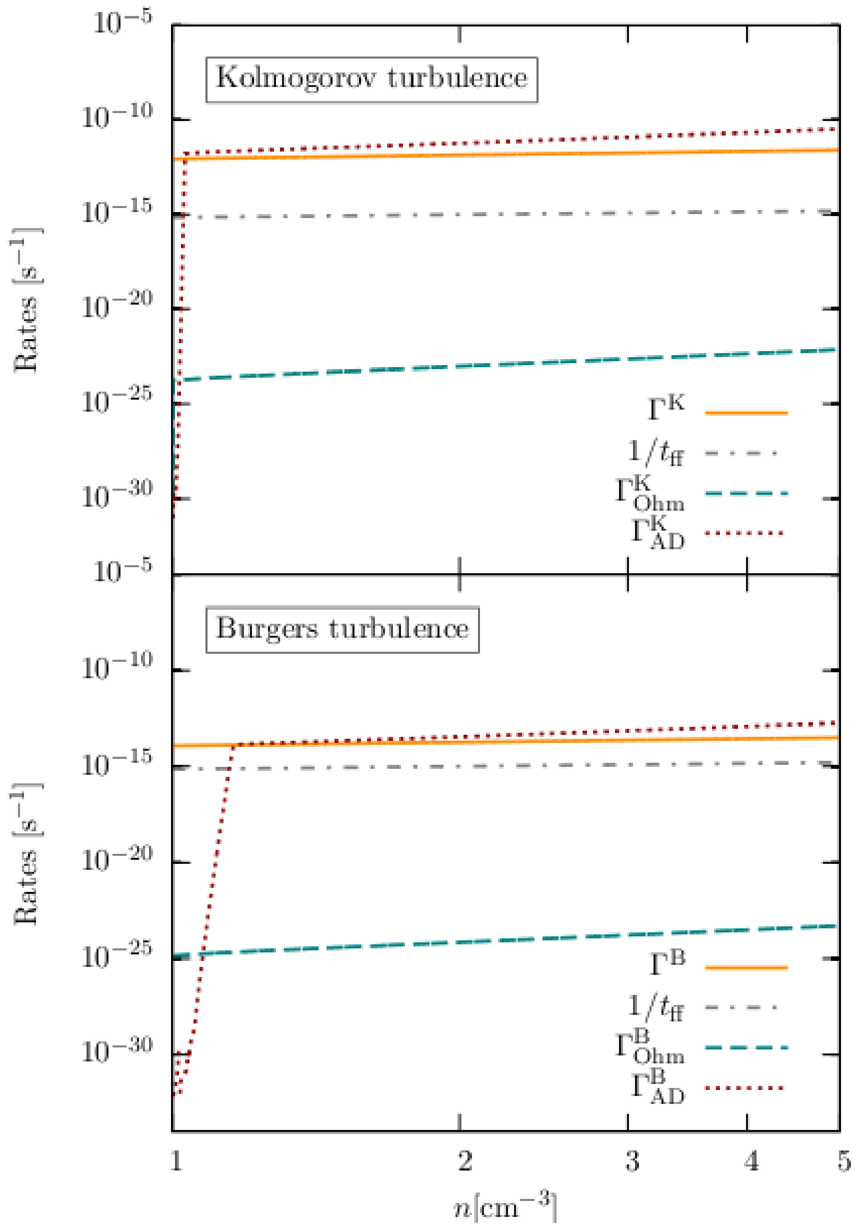
- growth rate for small Pm:

$$\Gamma \propto \mathrm{Rm}^{(1-\vartheta)/(1+\vartheta)}$$



Schober et al., submitted

Different Rates on Viscous Scale



- growth rate:

$$\Gamma^{\mathrm{K}} \propto \mathrm{Re}^{1/2}$$

$$\Gamma^{\mathrm{B}} \propto \mathrm{Re}^{1/3}$$

- diffusion rates:

$$\Gamma_{\mathrm{Ohm}} \approx \frac{\eta_{\mathrm{Ohm}}}{\ell^2}$$

$$\Gamma_{\mathrm{AD}} \approx \frac{\eta_{\mathrm{AD}}}{\ell^2}$$

- inverse free-fall time:

$$\frac{1}{t_{\mathrm{ff}}} = \left(\frac{3\pi}{32Gmn}\right)^{-1/2}$$

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Magnetic Energy on Viscous Scale

- **small-scale magnetic energy:**

$$\frac{\mathrm{d}E_\mathrm{B}}{\mathrm{d}t} = \left[\Gamma + \frac{4}{3n}\frac{\mathrm{d}n}{\mathrm{d}t} - \Gamma_\mathrm{Ohm} - \Gamma_\mathrm{AD}(E_\mathrm{B})\right] E_\mathrm{B}$$

growth rate of the small-scale dynamo

spherical gravitational compression

$$B \propto n^{2/3}$$

Ohmic dissipation

$$\Gamma_\mathrm{Ohm} \approx \frac{\eta_\mathrm{Ohm}}{\ell^2}$$

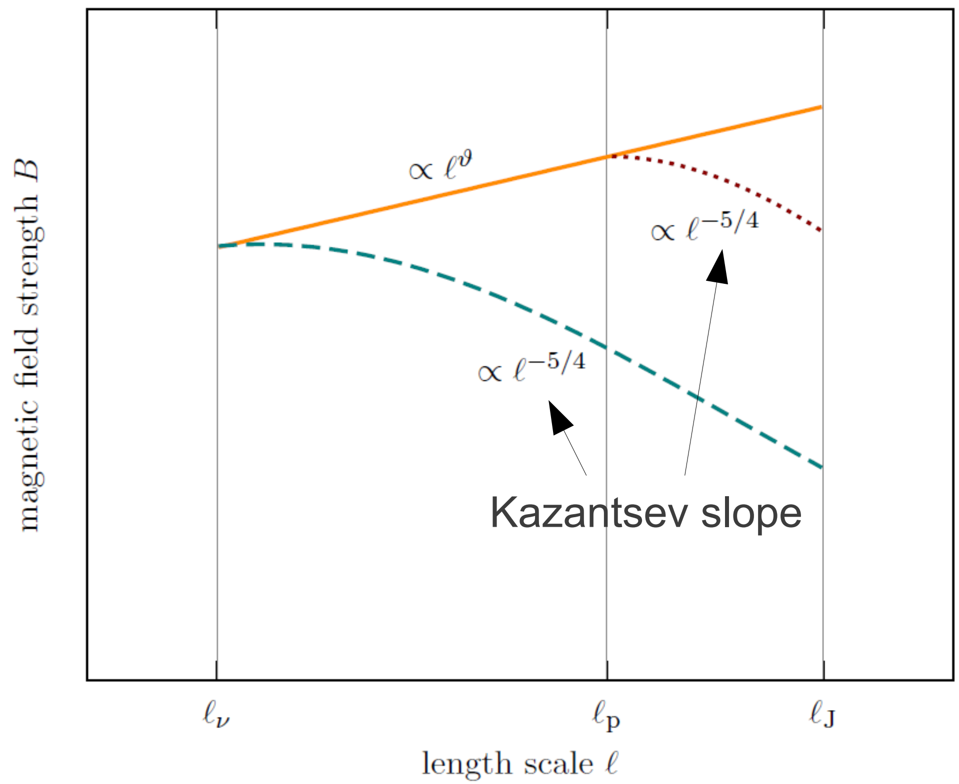
ambipolar diffusion

$$\Gamma_\mathrm{AD} \approx \frac{\eta_\mathrm{AD}}{\ell^2}$$

- **magnetic field strength:**

$$B = (8\pi E_B)^{1/2}$$

Transport to Larger Scales: Non-Linear Dynamo



- after saturation on viscous (small) scale peak moves to larger scales
- assumption: peak moves to larger scales on the eddy timescale [Schekochihin et al., New J. Phys., 2002]
- peak scale:

$$\ell_{\rm p}(t) = \ell_\nu(t_\nu) + \left(\frac{v_{\rm J}}{\ell_{\rm J}^{\vartheta}}\,(t - t_\nu)\right)^{1/(1-\vartheta)}$$

=> calculation of magnetic energy on the Jeans scale

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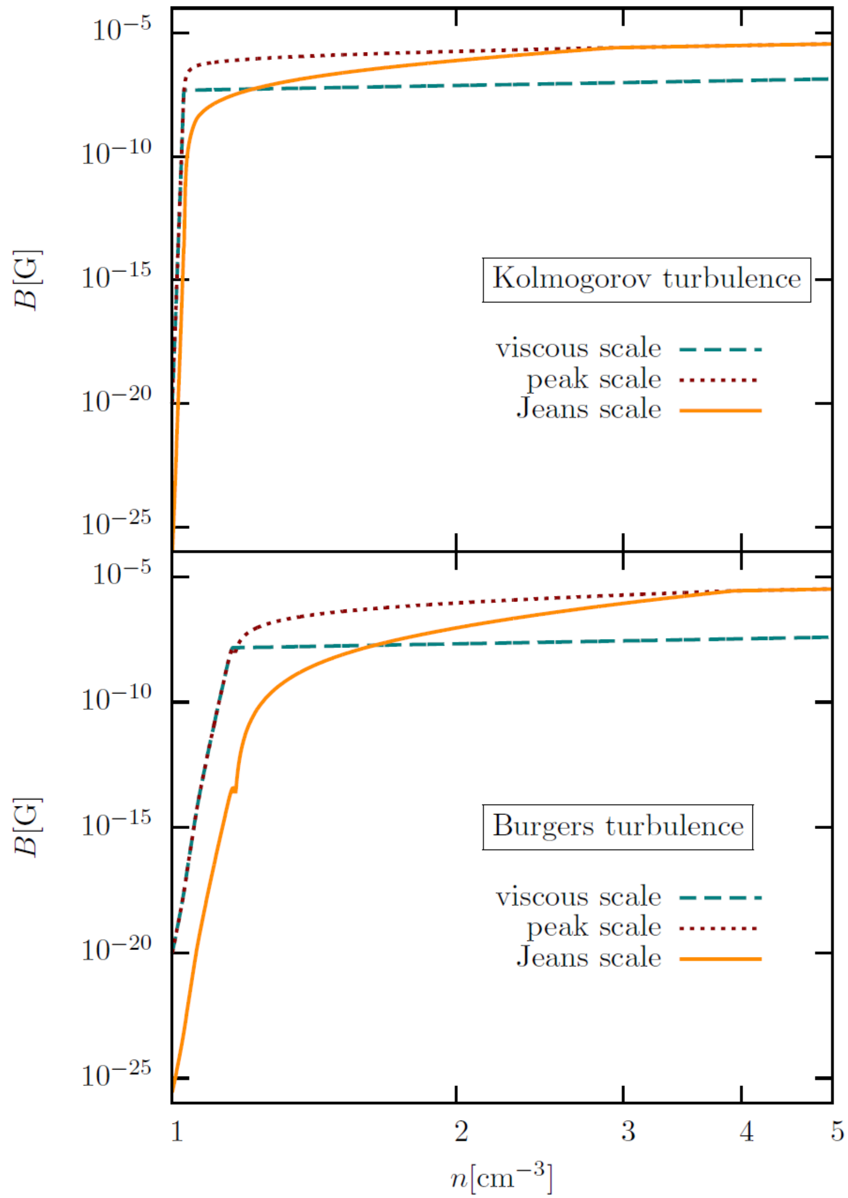
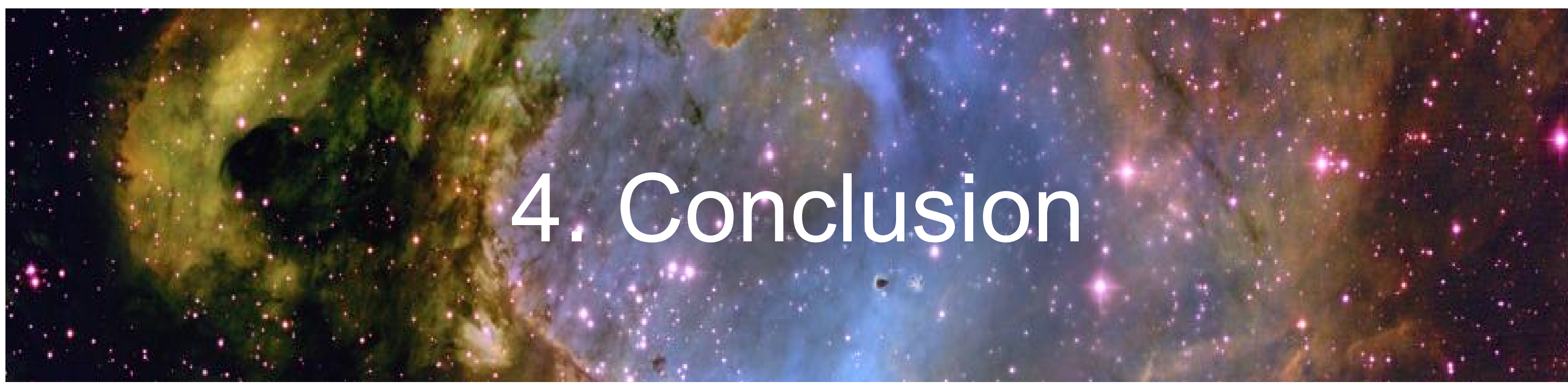
Magnetic Field Evolution



=> Magnetic field saturates almost instantly (in terms of density) for all types of turbulence !!!

[Schober et al., ApJ, 2012]


4. Conclusion

Conclusion

- **critical magnetic Reynolds number** for small-scale dynamo action:

  $\mathrm{Rm} > 110$ Kolmogorov turbulence ($\vartheta = 1/3$)

  $\mathrm{Rm} > 2700$ Burgers turbulence ($\vartheta = 1/2$)

- **growth rate** of magnetic energy:

  $\Gamma \propto \mathrm{Re}^{(1-\vartheta)/(1+\vartheta)}$ large magnetic Prandtl number

  $\Gamma \propto \mathrm{Rm}^{(1-\vartheta)/(1+\vartheta)}$ small magnetic Prandtl number

- small-scale dynamo in **primordial star formation**:

  Dynamical important magnetic fields can be generated on small timescales compared to the free-fall time.

Thanks for your attention!

Further questions/comments? →
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